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Solving the Thermal Economic Dispatch Problem Using MATLAB Optimization Tools

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Abstract

In this study, the thermal economic generation (dispatch) is solved using MATLAB optimization tools. First, the thermal economic dispatch problem is reviewed, concentrating on the λ-iteration method to solve the problem. The MATLAB optimization function *fmincon* is looked into describing its parameters and outputs. Utilization of *fmincon* to solve the thermal economic dispatch problem is detailed. The practical implementation includes IEEE published data system consisting of 26 units, with all the units committed. The second system consists of 10 units with a possibility of decommitting some of them. The results have proved the effectiveness of the MATLAB optimization tools to solve large-scale power systems problems.

Keywords: Thermal Economic Dispatch, Optimal Generation, Lambda Iteration Method, MATLAB Optimization Functions.

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حل مسألة الإرسال الإقتصادي الحراري باستخدام أدوات الحل الأمثل في الماتلاب

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الملخص
في هذه الدراسة تم حل مسألة مسألة التوليد (الإرسال) الإقتصادي الحراري باستخدام أدوات الحل الأمثل في الماتلاب. بداية تمت مراجعة مسألة الإرسال الإقتصادي الحراري مع التركيز على حلها باستخدام طريقة λ–التكرارية. ثم تمت مراجعة دالة الحل الأمثل في الماتلاب *fmincon* مع وصف معاملاتها ومخرجاتها. تم شرح كيفية استخدام الدالة *fmincon* لحل مسألة الإرسال الإقتصادي الحراري بالتفصيل. شمل التطبيق العملي منظومة قدرة منشورة البيانات في IEEE تتكون من 26 وحدة بحيث أن جميع الوحدات في حالة تشغيل. تتكون المنظومة الثانية من 10 وحدات مع إمكانية إطفاء بعض منها. أثبتت النتائج فاعلية أدوات الحل الأمثل في ماتلاب لحل مسائل منظومات القدرة كبيرة الحجم.

الكلمات المفتاحية: الإرسال الإقتصادي الحراري، التوليد الأمثل، طريقة λ–التكرارية، دوال الحل الأمثل في الماتلاب.

1. Introduction

The purpose of the economic dispatch problem is to minimize the generation cost required to supply a certain load while satisfying all the operating constraints. Thermal economic dispatch concerns with the fossil (oil, gas, coal, etc.)-fired units. It is an optimization problem with the objective (cost) function that is the hourly fuel cost and may include labor and maintenance costs. Solving the problem may lead to a saving of millions of dollars in fuel costs every year.

Many methods have been used to solve the problem, such as λ-iteration, Lagrange multipliers, Newton's method, Gradient-based techniques, Linear and Quadratic Programming, and artificial

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intelligent-based methods such as Artificial Neural Networks (ANNs). A literature review of the different methods to solve the economic dispatch problem can be found in [1]. To solve the thermal economic dispatch problem, the hourly fuel cost (in $/h) versus MW output data must be available for each unit in the system. This data is called input-output curve. There are many representations of these curves, such as quadratic, piecewise quadratic, and piecewise linear. The λ-iteration method with a quadratic input-output curve has been adopted in this study.

2. Mathematical Formulation of the Thermal Economic Dispatch Problem

With quadratic representation, the hourly fuel cost $\boldsymbol{F_i}$ ($/h) of unit $\boldsymbol{i}$ is [2]:

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (1)$$

Where: $\boldsymbol{P_i}$ is the output of unit $\boldsymbol{i}$ (MW), a_i is the constant coefficient of the cost function ($/h), $\boldsymbol{b_i}$ is the linear coefficient of the cost function ($/MWh), and $\boldsymbol{c_i}$ is the quadratic coefficient of the cost function ($/MW^{2}h).

The first derivative of the cost function is called the incremental hourly fuel cost ($/MWh):

$$\frac{dF_i}{dP_i} = b_i + 2c_i P_i \quad (2)$$

The objective (cost) function to be minimized is the sum of the fuel cost of all the committed units $\boldsymbol{F_T}$ ($/h):

$$F_T = \sum_{i=1}^{Nu} F_i(P_i) = \sum_{i=1}^{Nu} (a_i + b_i P_i + c_i P_i^2) \quad (3)$$

Subject to the following constraints:
- System (global) constraint: balance of generation and load:

$$\sum_{i=1}^{Nu} P_i = P_D \quad (4)$$

- Unit (local) constraint: generation limits of each unit:

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$$P_{min,i} \le P_i \le P_{max,i} \quad (5)$$

Where: $\boldsymbol{Nu}$ is the total number of committed units, $\boldsymbol{P_D}$ is the load (demand), $\boldsymbol{P_{min,i}}$ is the minimum allowable output of unit $\boldsymbol{i}$, and $\boldsymbol{P_{max,i}}$ is the maximum allowable output of unit $\boldsymbol{i}$.
It is required to find the optimal outputs of all committed units which lead to minimum fuel cost; that is:

$$\text{minimize } F_T(P_i) = \text{minimize } \sum_{i=1}^{Nu}(a_i + b_i P_i + c_i P_i^2) \quad (6)$$

Subject to global and local constraints. Note that the system constraint assumes that the thermal units are connected directly to the load neglecting the transmission loss as shown in Figure 1. Also, the spinning reserve requirements are ignored.

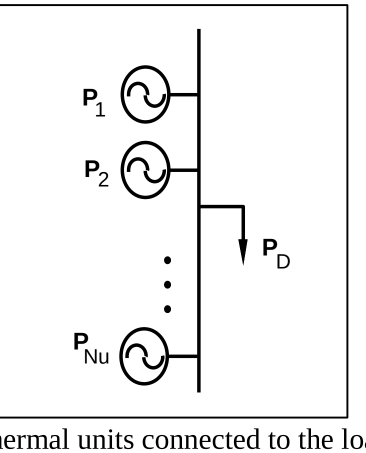

Figure 1. Thermal units connected to the load directly

3. The λ-Iteration Method

The hourly fuel cost function can be optimized (minimized in our case) using the optimization calculus. Consequently, the Lagrangian multiplier technique is used. The necessary steps for optimization will be summarized below [2].
The Lagrangian function $\boldsymbol{\mathcal{L}}$ is formed by appending the system constraint to the cost function using the Lagrange multiplier $\boldsymbol{\lambda}$:

$$\mathcal{L}(P_i, \lambda) = F_T + \lambda\left(P_D - \sum_{i=1}^{N_u} P_i\right) \quad (7)$$

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The total hourly fuel cost $\boldsymbol{F_T}$ can be minimized by taking the first derivative of $\boldsymbol{\mathcal{L}}$ with respect to all $\boldsymbol{P_i}$ and $\boldsymbol{\lambda}$, equating the resulting functions to zero, and solving the resulting equations to obtain the optimal values of $\boldsymbol{P_i}$ and $\boldsymbol{\lambda}$.

$$\frac{d\mathcal{L}(P_i,\lambda)}{dP_i} = \frac{d}{dP_i}\left[F_T + \lambda\left(P_D - \sum_{i=1}^{N_u} P_i\right)\right] = 0 \tag{8}$$

$$\rightarrow \frac{dF_i}{dP_i} = b_i + 2c_iP_i = \lambda \tag{9}$$

That is, the incremental hourly fuel costs of all committed units should be equal, at which the Lagrange multiplier is the optimal one ($\boldsymbol{\lambda_{optimal}}$) and the power output values are optimal ($\boldsymbol{P_{i,optimal}}$).
Differentiating $\boldsymbol{\mathcal{L}}$ with respect to $\boldsymbol{\lambda}$ and equating to zero results in the system constraint itself.
Given a certain load $\boldsymbol{P_D}$, it is required to find $\boldsymbol{\lambda_{optimal}}$ from which $\boldsymbol{P_{i,optimal}}$ can be obtained using eq. (9). The question is: how to find $\boldsymbol{\lambda_{optimal}}$?
The $\boldsymbol{\lambda}$-iteration method is an effective way to find $\boldsymbol{\lambda_{optimal}}$. In this method an initial value of $\boldsymbol{\lambda}$ is estimated from which the output power of all committed units is computed as follows:

$$P_i = \frac{\lambda - b_i}{2c_i} \tag{10}$$

If $\boldsymbol{P_i > P_{max,i}}$ then $\boldsymbol{P_i}$ is set equal to $\boldsymbol{P_{max,i}}$, and if $\boldsymbol{P_i < P_{min,i}}$ then $\boldsymbol{P_i}$ is set equal to $\boldsymbol{P_{min,i}}$.
Then $\boldsymbol{\lambda}$ is corrected depending on the difference (error) between the demand and sum of generation:

$$\epsilon = P_D - \sum_{i=1}^{Nu} P_i \tag{11}$$

The process is repeated until $\boldsymbol{\epsilon}$ falls below a certain specified value. To obtain a feasible solution using the $\boldsymbol{\lambda}$-iteration method the load value must lie within the sum of limits of all the committed unit, that is:

$$\sum_{i=1}^{Nu} P_{min,i} \le P_D \le \sum_{i=1}^{Nu} P_{max,i} \tag{12}$$

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So, if $\boldsymbol{P_D} < \sum_{i=1}^{Nu} \boldsymbol{P_{min,i}}$, then some units have to be switched off (decommitted), and if $\boldsymbol{P_D} > \sum_{i=1}^{Nu} \boldsymbol{P_{max,i}}$, then more units have to be brought online (committed).
The process of correcting λ from iteration to iteration can be carried on by different methods, such as interpolation and binary search. In this study binary search method is used. A good estimation of the initial $\boldsymbol{\lambda}$ is important to ensure convergence of the solution.
Finally, it is to be noted that in some cases, specifically when the input-output curve is not quadratic, the λ-iteration method cannot solve the thermal economic dispatch problem, and other methods have to be used.

4. The MATLAB Optimization Function *fmincon*

MATLAB provides a variety of powerful tools and functions to solve the different kinds of optimization problems: linear, nonlinear, unconstrained, constrained, etc. Some of these functions are:
- *fminbnd*: finds the minimum of a single-variable function on a fixed interval.
- *fminsearch*: finds the minimum of an unconstrained multivariable function using a derivative-free method.
- *fminunc*: finds the minimum of an unconstrained multivariable function.
- *fmincon*: finds the minimum of a constrained nonlinear multivariable function.
- *linprog*: solves linear programming problems.
- *quadprog*: solves quadratic programming.
- *intlinprog*: solves mixed-integer linear programming.

All the above functions aim to minimize a cost function *f(x)* and find the optimal independent variables *x*; that is, $minimize_x \ f(x)$ subject to the different constraints.

In this study, the function *fmincon* is used to solve the thermal economic dispatch. This function has many forms with different parameters and outputs, such as [3],
[*x*, *fval*] = *fmincon* (*fun*, *x0*, *A*, *b*, *Aeq*, *beq*, *lb*, *ub*, *nonlcon*)
Where:
fun: objective (cost) function, *f(x)*.
x0: estimated initial values of *x*.
A, *b*: constants of the linear inequalities, $A \cdot x \leq b$.

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Aeq, *beq*: constants of the linear equalities, $Aeq \,.\, x = beq$.
lb, *ub*: lower and upper bounds of *x*, $lb \leq x \leq ub$.
nonlcon: functions representing nonlinear inequalities and equalities constraints, $c(x) \leq 0$ & $ceq(x) = 0$.

The outputs of *fmincon* (return values):
x: resulting optimal *x*.
fval: resulting minimized cost function.
The MATLAB help documentation should be consulted for detailed implementation of *fmincon*.

5. Solving the Thermal Economic Dispatch Problem Using *fmincon*

To demonstrate the utilization of *fmincon* to solve the thermal economic dispatch, a small system consisting of three thermal units is used [2]. Data of the system is given below:-

Unit 1: $F_1(P_1) = 561 + 7.92P_1 + 0.001562P_1^2$ \$/h,
$150 \text{ MW} \leq P_1 \leq 600 \text{ MW}$
Unit 2: $F_2(P_2) = 310 + 7.85P_2 + 0.00194P_2^2$ \$/h,
$100 \text{ MW} \leq P_2 \leq 400 \text{ MW}$
Unit 3: $F_3(P_3) = 78 + 7.97P_3 + 0.00482P_3^2$ \$/h,
$50 \text{ MW} \leq P_3 \leq 200 \text{ MW}$
Sum of minimum outputs $\sum_{i=1}^{3} P_{min,i} = 300$ MW
Sum of maximum outputs $\sum_{i=1}^{3} P_{max,i} = 1200$ MW

These units are committed to supply a load of $P_D = 850$ MW.
It is well known that starting with good initial values of the independent variables (MW outputs in this case) is crucial to find optimal solution. However, for the current example, the initial power outputs will be given random values that satisfy both unit and system constraints. The chosen initial values are,

$P_{0,1} = 450$ MW, $P_{0,2} = 300$ MW, and $P_{0,3} = 100$ MW.

The MATLAB code is given below:

```
a = [561; 310; 78];
b = [7.92; 7.85; 7.97];
c = [0.001562; 0.00194; 0.00482];
Pmin = [150; 100; 50];
Pmax = [600; 400; 200];
```

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```
PD = 850;
P0 = [450; 300; 100];
FT = @(P) sum(a + b.*P + c.*P.^2);
[Popt, FTopt] = fmincon(FT, P0, [], [], [1 1 1],
PD, Pmin, Pmax, [])
```

Where:
Popt: array of optimal outputs (MW).
FTopt: optimal (minimum) hourly fuel cost ($/h).
Note that the empty square brackets in the parameters of *fmincon* indicate that there are no corresponding constraints.

Executing the above code gives the following results:

```
Popt =
  393.1699
  334.6038
  122.2264
FTopt =
   8.1944e+03
```

Hence, the minimum fuel cost to supply the 850 MW load is 8194.4 $/h, and this minimum cost is achieved when P_1 =393.2 MW, P_2 = 334.6 MW, and P_3 = 122.2 MW. This result conforms exactly with the results obtained in [2].

6. Test Results and Discussion

The function *fmincon* will be applied to a more practical system; that is, the 26 thermal generating units system of reference [4]. Data of the system is given in Table 1.

Table 1. 26 generating units' data

Unit	P_{max} (MW)	P_{min} (MW)	a ($/h)	b ($/MWh)	C ($/MW^{2}h)
1	12	2.4	24.3891	25.5472	0.02533
2	12	2.4	24.4110	25.6753	0.02649
3	12	2.4	24.6382	25.8027	0.02801
4	12	2.4	24.7605	25.9318	0.02842
5	12	2.4	24.8882	26.0611	0.02855
6	20	4	117.7551	37.5510	0.01199
7	20	4	118.1083	37.6637	0.01261
8	20	4	118.4576	37.7770	0.01359
9	20	4	118.8206	37.8896	0.01433

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10	76	15.2	81.1364	13.3272	0.00876
11	76	15.2	81.2980	13.3538	0.00895
12	76	15.2	81.4641	13.3805	0.00910
13	76	15.2	81.6259	13.4073	0.00932
14	100	25	217.8952	18.0000	0.00623
15	100	25	218.3350	18.1000	0.00612
16	100	25	218.7752	18.2000	0.00598
17	155	54.25	142.7348	10.694	0.00463
18	155	54.25	143.0288	10.7154	0.00473
19	155	54.25	143.3179	10.7367	0.00481
20	155	54.25	143.5972	10.7583	0.00487
21	197	68.95	259.1310	23.0000	0.00259
22	197	68.95	259.6490	23.1000	0.00260
23	197	68.95	260.1760	23.2000	0.00263
24	350	140	177.0575	10.8616	0.00153
25	400	100	310.0021	7.4921	0.00194
26	400	100	311.9102	7.5031	0.00195

For this system $\sum_{i=1}^{26} P_{min,i} = 927.65$ MW and $\sum_{i=1}^{26} P_{max,i} = 3105$ MW. It is assumed that all the 26 units are committed, so the load should fall between those two values.
Three load values are tested; 1200 MW, 2016 MW, and 3000 MW.
Initial values of units' outputs are set according to the following algorithm: First, each unit is loaded to its minimum output, then the remaining load is distributed between the units according to their difference between maximum and minimum outputs; that is:

$$P_{0,i} = P_{min,i} + \frac{(P_{max,i} - P_{min,i})}{(\sum_{i=1}^{Nu} P_{max,i} - \sum_{i=1}^{Nu} P_{min,i})} \left(P_D - \sum_{i=1}^{Nu} P_{min,i}\right) \quad (13)$$

Applying the above formula ensures that the initial estimated outputs satisfy the system constraint.
The results are given in Table 2.

Table 2 Results of the 26 units test system

P_i (MW)	**P_D**		
	1200 MW	**2016 MW**	**3000 MW**
P_1	2.40	2.40	7.27
P_2	2.40	2.40	4.53
P_3	2.40	2.40	2.40
P_4	2.40	2.40	2.40
P_5	2.40	2.40	2.40

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P_6	4.00	4.00	4.00
P_7	4.00	4.00	4.00
P_8	4.00	4.00	4.00
P_9	4.00	4.00	4.00
P_{10}	15.19	15.20	76.00
P_{11}	15.20	15.20	76.00
P_{12}	15.20	15.20	76.00
P_{13}	15.20	15.20	76.00
P_{14}	25.00	25.00	100.00
P_{15}	25.00	25.00	100.00
P_{16}	25.00	25.00	100.00
P_{17}	54.25	132.03	155.00
P_{18}	54.25	126.98	155.00
P_{19}	54.25	122.65	155.00
P_{20}	54.25	118.92	155.00
P_{21}	68.95	68.95	197.00
P_{22}	68.95	68.95	197.00
P_{23}	68.95	68.95	197.00
P_{24}	140.00	344.77	350.00
P_{25}	238.20	400.00	400.00
P_{26}	234.15	400.00	400.00
$\sum_{i=1}^{26} P_i$	1200	2016	3000
F_T (\$/h)	19340	27862	46496

There is a perfect conformation between results obtained using *fmincon* and results obtained using the developed code based on the λ-iteration method in reference [5].

6.1 Decommitting Some Units

In practice, when the load decreases then committing all units becomes uneconomical because some units have to operate at their minimum output, therefore, it would be more economical to decommit these units. To determine which units must be committed (ON) and which decommitted (OFF) an exhaustive search method can be used. In this method all the possible ON/OFF combinations have to be examined to determine the combination with the lowest fuel cost. Since each unit can be either ON or OFF, there would be 2^{Nu} combinations for Nu committed units. Of course, many of these combinations are infeasible because they violate the required constraints; that is, $\sum_{ON\ units} P_{max,i} < P_D$.

A new code is added to the previous code to include the exhaustive enumeration search for the ON/OFF combination with lowest cost.

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Since applying exhaustive search to the 26 units system would result in examining more than 67×10^6 of combination which is impractical, then another system consists of 10 thermal units is used as a test system (1024 combinations). This system is a subsystem of the 26 units system. It includes the following units: 10-13, 17-20, and 25-26. For this system $\sum_{i=1}^{10} P_{min,i} = 477.8$ MW and $\sum_{i=1}^{10} P_{max,i} = 1724$ MW. Two load values are tested, 749.6 MW and 1321.5 MW. For each load the economic dispatch solution using *fmincon* is carried twice, once with all units committed, and then with applying exhaustive search to find the optimal ON/OFF combination. The results are given in Table 3.

Table 3. Results of the 10 units test system

	P_{min} (MW)	P_{max} (MW)	P_D = 749.6 MW		P_D = 1321.5 MW	
			All ON	ON/OFF	All ON	ON/OFF
P_1	15.20	76	15.20	OFF	15.20	OFF
P_2	15.20	76	15.20	OFF	15.20	OFF
P_3	15.20	76	15.20	OFF	15.20	OFF
P_4	15.20	76	15.20	OFF	15.20	OFF
P_5	54.25	155	54.25	OFF	121.78	137.41
P_6	54.25	155	54.25	OFF	116.95	132.24
P_7	54.25	155	54.25	OFF	112.79	127.82
P_8	54.25	155	54.25	OFF	109.18	124.03
P_9	100	400	237.92	377.18	400.00	400.00
P_{10}	100	400	233.88	372.42	400.00	400.00
$\sum_{i=1}^{10} P_i$	477.8	1724	749.6	749.6	1321.5	1321.5
F_T (\$/h)	-	-	8478.61	6788.54	14155.33	13731.89
% F_T Saving	-	-	Reference	-19.93%	Reference	-2.99%
Average CPU run time*	-	-	0.01 s	7.58 s	0.02 s	1.25 s

*CPU used: Intel Core i7-6820HQ CPU @2.70 GHz, 16 GB DDR4 RAM.

From the above results, it is clear that committing all units in the case of a low load of 749.6 MW (which is slightly above $\sum_{i=1}^{10} P_{min,i}$) necessitates fixing outputs of units 1-8 at minimum. This makes the dispatch to be far from being optimal. In contrast, by allowing some units to be decommitted, then only the two largest units are required to be committed. In this case, a notable fuel saving of about 20% has been achieved. Even in the case of a high load of

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1321.5 MW (which is slightly less than $\sum_{i=1}^{10} P_{max,i}$), it is more economical to decommit some units and gain some fuel cost saving as shown in Table 3.
The average CPU run time has been obtained by repeating the execution of the code 10 times and taking the average run time. It is expected that the run time would increase exponentially by increasing the number of units in the case of exhaustive search.

7. **Conclusion**

Using MATLAB optimization tools has the great advantage of being able to utilize a powerful tool that can handle a wide variety of optimization problems. In this study the MATLAB optimization function *fmincon* has been utilized to solve an important problem in the electric power systems; that is, the thermal economic dispatch problem with basic constraints. The challenge was how to adopt the cost function and the constraints to fit in the parameters of *fmincon*, which necessitated a deep study of it.
The test system has included 26 thermal units assuming all units are committed. The results have proved a complete conformation with the results obtained from a previous study that used the λ-iteration method.
Then decommitting some units aiming to achieve more optimal solution has been studied. This requires exhaustive search by examining all ON/OFF unit combinations and solve the problem for each combination. For this case the test system has included 10 thermal units which necessitates examining 1024 different combinations. A code has been added to generate all the possible ON/OFF combinations and execute *fmincon* for each feasible (satisfying constraints) combination. The optimal combination has been found in a reasonably short time.
In summary, this study has proved the suitability and effectiveness of MATLAB optimization tools to solve practical problems in the field of electrical power systems.

Recommendations

- Inclusion of more operational constraints, such as spinning reserve requirement and transmission losses.
- Testing the decommitting policy with larger systems. This may require using a more powerful computer.

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- Investigation of utilizing the MATLAB optimization tools to solve the economic dispatch problem for power system with different input-output curve representations, and non-thermal units such as hydro-units.
- Expanding the study to incorporate the unit commitment problem which searches for the optimal generation strategy over a definite time period.

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